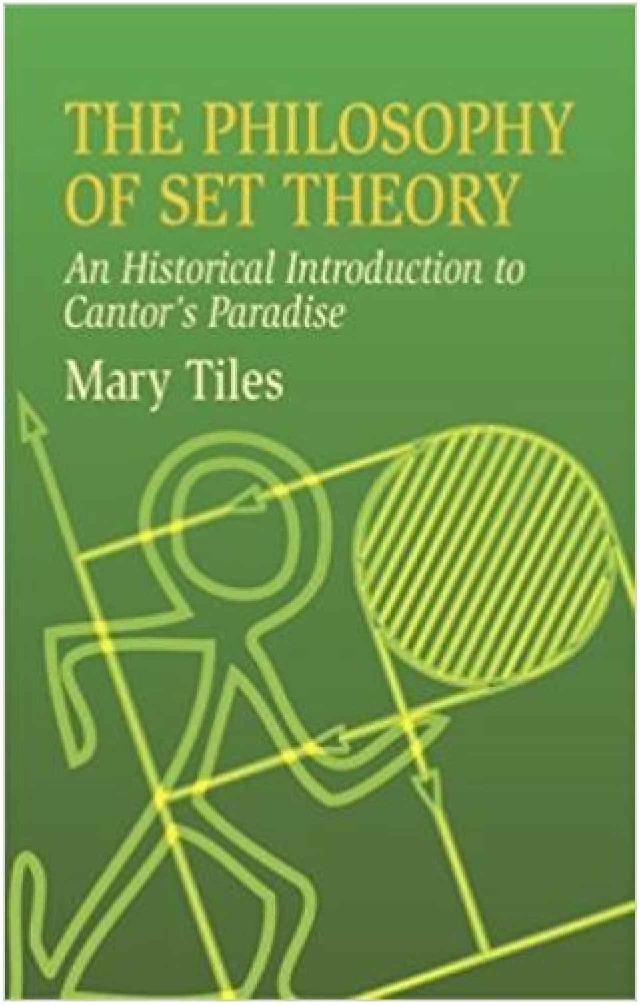
The Philosophy Of Set Theory: An Historical Introduction To Cantor's Paradise (Dover Books On Mathematics)



Synopsis

A century ago, Georg Cantor demonstrated the possibility of a series of transfinite infinite numbers. His methods, unorthodox for the time, enabled him to derive theorems that established a mathematical reality for a hierarchy of infinities. Cantor’s innovation was opposed, and ignored, by the establishment; years later, the value of his work was recognized and appreciated as a landmark in mathematical thought, forming the beginning of set theory and the foundation for most of contemporary mathematics.As Cantor’s sometime collaborator, David Hilbert, remarked, "No one will drive us from the paradise that Cantor has created." This volume offers a guided tour of modern mathematics’ Garden of Eden, beginning with perspectives on the finite universe and classes and Aristotelian logic. Author Mary Tiles further examines permutations, combinations, and infinite cardinalities; numbering the continuum; Cantor’s transfinite paradise; axiomatic set theory; logical objects and logical types; and independence results and the universe of sets. She concludes with views of the constructs and reality of mathematical structure.Philosophers with only a basic grounding in mathematics, as well as mathematicians who have taken only an introductory course in philosophy, will find an abundance of intriguing topics in this text, which is appropriate for undergraduate-and graduate-level courses.

Book Information

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Customer Reviews

Excellent book! Having some background in mathematics and philosophy will definitely help. I have been interested in these topics since the seventies and have never found a book as informative and interesting as this! I have found good philosophical books that were mathematically deficient and good mathematical books that were philosophically naÃƒÆ’Ã Â ve. This book is a rare find!

This book is an exploration of the philosophic consequences of the infinite, both potential and actual. It starts out looking at ZenoÃƒÂ¢Ã â ¬Ã â„¢s paradoxes. From there AristotleÃƒÂ¢Ã â ¬Ã â„¢s view is look at. Then, limits are examine and what they mean for the finitist. Next the continuum is introduce and CantorÃƒÂ¢Ã â ¬Ã â„¢s continuum hypothesis that was posited under his study of transfinite numbers. Axiomatic set theory comes on the scene, and after this the logicistsÃƒÂ¢Ã â ¬Ã â„¢ program is discussed. Finally, the attempts to solve the continuum hypothesis and the independence results and the axiom of choiceÃƒÂ¢Ã â ¬Ã â„¢s roll is presented. The last chapter attempts to provide some sense of a resolution of the status of the infiniteÃƒÂ¢Ã â ¬Ã â œits necessity from both a realist and a constructivist point of view.Here are some of the my comments I thought interesting. (Pagination is from the Kindle edition)[page 3] In discussing the opposing viewpoints whether mathematics is discovered or invented, Mary Tiles writes: ÃƒÂ¢Ã â ¬Ã Å“It may be found that one of these alternatives is to be preferred, or that different alternatives are useful for different purposes.ÃƒÂ¢Ã â ¬Ã Â• Upon reading this I thought that pragmatism is alive and well even in mathematics.[page 8] After discussing the view that mathematics needs to be linked to some empirical content, she states: ÃƒÂ¢Ã â ¬Ã Å“Following this line of argument, some empiricists have been led to conclude that there is no sense to be given to such talk [of the actual infinite].ÃƒÂ¢Ã â ¬Ã Â• I thought what would the pure mathematicians have to say of such a view of mathematics.[page 9] ÃƒÂ¢Ã â ¬Ã Å“But what case can the realist make which might persuade the finitist (an anti-realist about the infinite), motivated by empiricism, of the error of his ways?ÃƒÂ¢Ã â ¬Ã Â• My answer is that one does not need realism or empiricism if mathematics is created by the human mind/brain whether or not it has any connection to the world. Once created mathematics takes on an objective status or at the very least, a inter-subjective one. Further on ÃƒÂ¢Ã â ¬Ã Å“. . . for space and time are presumed to be continuous.ÃƒÂ¢Ã â ¬Ã Â• From my understanding of quantum mechanics space and time may actually be discrete. Lastly, ÃƒÂ¢Ã â ¬Ã Å“. . . since neither space or time can coherently be thought to have a boundary . . .ÃƒÂ¢Ã â ¬Ã Â• Again science intervenes. Under general relativity the universe has a boundary, even though we may not be able to see it, being outside our locationÃƒÂ¢Ã â ¬Ã â„¢s light-cone. This is so even for the ever expanding universe, which we very well may be living in. Nothing lies outside this expanding boundary of the universe.[page 20] In discussing AristotleÃƒÂ¢Ã â ¬Ã â„¢s response to ZenoÃƒÂ¢Ã â ¬Ã â„¢s paradoxes, somewhat technically, she states:

Ãƒâ¢Ã â ¬Ã Å“Where is M at t? It seems that either M does not have a position at t or that it is in two positions at t, which violates the assumption that material objects occupy exactly one place at one time and occupy some place at all times.Ãƒâ¢Ã â ¬Ã Â• These are no violations according to quantum mechanics description of atomic particles and events.[page 22] She asks: Ãƒâ¢Ã â ¬Ã Å“Can that all-embracing whole, the physical world, or the universe, be anything short of infinite?Ãƒâ¢Ã â ¬Ã Â• Most definitely yes (see previous comments).[page 47] According to the nominalists: Ãƒâ¢Ã â ¬Ã Å“All classification is an imposition by the human mind (whether this is as a product of human nature, as in Ockham . . . or of human convention in defining words, as in Hobbes . . .)Ãƒâ¢Ã â ¬Ã Â• This fits in with my notion that it is us humans who give meaning to the world. I guess this makes me a nominalist. Platonic realism is a shadow on the wall in the cave, not the Sun casting the shadows.[page 129] Another example of possible pragmatism in mathematics: Ãƒâ¢Ã â ¬Ã Å“The question regarding the adoption of further axioms may be complicated, but is basically a question of what will be useful or what is required by other areas of mathematics.Ãƒâ¢Ã â ¬Ã Â•[page 195] After asking whether the continuum hypothesis should be considered true or false, she asks further: Ãƒâ¢Ã â ¬Ã Å“Should one not perhaps conclude that there are several set-theoretic structures, each of which can legitimately be explored by the mathematician?Ãƒâ¢Ã â ¬Ã Â• If this attitude should be taken that leaves a big hole in foundationalism, at least in connection with the hope that set theory was to provide such a foundation.[page 208-9] Ãƒâ¢Ã â ¬Ã Å“One might find some definitions more useful than others, but ultimately utility will be judged by reference to non-mathematical applications, not by application strictly within mathematics.Ãƒâ¢Ã â ¬Ã Â• Once again she brings in pragmatism, though she never mentions the term in the book. I disagree with her here (not her pragmatism). If mathematics is a creative discipline, than any system created is valid as long as it is consistent. Think of art for artÃƒâ¢Ã â ¬Ã â„¢s sake.I must admit right off the bat that I got lost in the sauce of technicalities making my way through the book, but there was just enough philosophy to keep me from bagging it. I probably skimmed through half of the more technical parts of the book. I did enjoy the philosophy parts of the book, and a bit of the easier technical material. I found that Mary Tiles might have ignore some of the necessary physics in her discussion of space and time. And while I am not certain, she seems to favor a pragmatic approach to set theory and possibly the rest of mathematics, which in my limited experience in philosophy of mathematics I have not found all that much of. I find such an approach to be reasonable. This maybe connected with my anti-realism (PlatoÃƒâ¢Ã â ¬Ã â„¢s kind), but I will spare the reader my critique of this.If, you are up to the challenge of technical and philosophical exploration of the infinite, the continuum hypothesis, and set theory, you may find the book interesting. If you are naive to any of these topics, it is definitely not a book for you. If, you lay somewhere in between like me, it may be a fifty-fifty proposition.

The Philosophy of Set Theory - An Historical Introduction to Cantor's Paradise by Mary Tiles is a fascinating mix of mathematics, mathematical logic, and philosophy that should appeal to (and challenge) both mathematics and philosophy majors at the undergraduate and graduate level.The focus is on the Generalized Continuum Hypothesis (GCH); the reader will meet topics like numbering the continuum, developing Cantor's transfinite ordinal and cardinal numbers, evaluating the ZF axioms underlying set theory, and examining the work of Frege and Russell.The first four chapters (The Finite Universe; Classes and Aristotelian Logic; Permutations, Combinations, and Infinite Cardinalities; and Numbering the Continuum) provide a historical, philosophical, and mathematical context for the more challenging chapters that follow. Some readers may wish to skip familiar sections although I found these early chapters to be quite engaging.Chapter 5 - Cantor's Transfinite Paradise is a good, standalone introduction to Cantor's transfinite ordinal and cardinal numbers and to the General Continuum Hypothesis (GCH).Chapter 6 - Axiomatic Set Theory is another good standalone chapter. Mary Tiles introduces the Zermelo-Fraenkel axioms that underlie modern set theory and develops a restatement of the GCH in the language of the ZF axioms.Chapter 7 - Logical Objects and Logical Types delves deeply into the work of Frege and Russell. This was not the first time that I had encountered Russell's ramified type hierarchy, but nonetheless I still found this section slow going.Chapter 8 - Independence Results and the Universe of Sets assumes substantial familiarity with model theory. Specific topics include Godel's constructible sets, cardinals and ordinals in models, inner models, and generic sets. Readers can either browse this technical chapter or omit it if they are willing to accept on trust the independence of the generalized continuum hypothesis and of the axiom of choice from the remaining Zermelo-Fraenkel set theory.The final chapter, Mathematical Structure - Construct and Reality, summarizes the key philosophic issues underlying not only the generalized continuum hypothesis, but also with set theory in general and with the theory of transfinite numbers in particular.I thoroughly enjoyed this introduction to Cantor's transfinite numbers. Mary Tiles has created an intriguing examination of the generalized continuum hypothesis.

The Philosophy of Set Theory: An Historical Introduction to Cantor's Paradise (Dover Books on Mathematics) Georg Cantor: His Mathematics and Philosophy of the Infinite Manifesto for Philosophy: Followed by Two Essays: "the (Re)Turn of Philosophy Itself" and "Definition of

Philosophy" (Suny Series, Intersections, Philosophy and Critical Theory) Satan, Cantor and Infinity: Mind-Boggling Puzzles (Dover Recreational Math) Songs and Prayers from Taize: Accompaniment Edition for Cantor & Instruments Naive Set Theory (Dover Books on Mathematics) Set Theory and Logic (Dover Books on Mathematics) A Book of Set Theory (Dover Books on Mathematics) An Outline of Set Theory (Dover Books on Mathematics) Axiomatic Set Theory (Dover Books on Mathematics) Set Theory and the Continuum Hypothesis (Dover Books on Mathematics) Set Theory and the Continuum Problem (Dover Books on Mathematics) Elementary Number Theory: Second Edition (Dover Books on Mathematics) 2nd (second) Edition by Underwood Dudley published by Dover Publications (2008) Introduction to Graph Theory (Dover Books on Mathematics) Game Theory: A Nontechnical Introduction (Dover Books on Mathematics) Pearls in Graph Theory: A Comprehensive Introduction (Dover Books on Mathematics) Introduction to Hilbert Space and the Theory of Spectral Multiplicity: Second Edition (Dover Books on Mathematics) Introduction to Modern Algebra and Matrix Theory: Second Edition (Dover Books on Mathematics) An Introduction to the Theory of Groups (Dover Books on Mathematics) Set Theory and Its Philosophy: A Critical Introduction